Abstract—This paper provides an attempt to apply a formal method (Montague grammar) to Arabic, as a pre-step towards its computation. Since semantic representation has to be compositional on the level of semantic processing, formalization based on Montague grammar can be utilized as a helpful and practical technique for the semantic construction of Arabic in Arabic understanding systems. As efforts of Arabic syntactic-semantic formalization are very limited, I hope that this approach might be a further motivation to redirect research to modern formal interpretation techniques for developing an adequate model of semantic processing for Arabic.

Index Terms—Arabic NLP, formalization of Arabic, Montague grammar, syntax-semantics connectivity.

I. INTRODUCTION

Many logicians and philosophers of language such as Rudolf Carnap [1]-[3], David Lewis [4], Saul Kripke [5], Richard Montague [6]-[8], Donald Davidson [9], among others, strongly believed in the capacity of formal languages and the applicability of formal methods to formalize and then interpret natural languages. In light of the hypothesis that natural languages can be formalized (i.e., amenable to receive formal interpretation, using formal languages and tools of logic) they positively applied their formal methods to different natural language groups. Successful syntactic-semantic formalization of different languages, such as English, Japanese and Thai, has led to promising achievements in the field of natural language processing (NLP).

Regarding to Arabic, my research did not discover any relevant logical literature on the question of how Arabic can be formally treated. That is, for Arabic, it was noted that the relationship of logic to language was an area much discussed in the medieval Arabic Aristotelian tradition (see [10], [11]), so the question has a historical pedigree, albeit one lacking continuity to the present. Therefore, the current work can be regarded as rebuilding the relationship between logic and Arabic language.

Moreover, for the last four decades concentration on Arabic formalisms and computation has focused on Arabic from the morphological and syntactical point of view (e.g., [12]-[16]). In this field, some success has been achieved. However, the importance of semantic processing for achieving the understanding capability, there were little works reported on semantic representation and semantic analysis of Arabic (17]-[20]). Therefore, I believe that there is an elemental need to make more effort to develop an adequate model for semantic processing for Arabic and even no existing formal theory is capable to provide a complete and consistent account of all phenomena involved in Arabic semantic processing.

This paper is an attempt to provide a syntactic-semantic formalization of a fragment of Modern Standard Arabic (MSA) as a pre-step towards its computation. I do so using Montague grammar (henceforth MG) as a formal method. MG is a well-known mathematically motivated logical method that treats natural languages semantic and of its relation with syntax. In general, the syntactic component of MG is formulated in terms of a categorial system. The semantic component of MG is formulated indirectly in terms of model-theoretic semantics via an intermediate language of intensional logic.

Based on the widespread applicability of MG to variety of natural languages such as: English, Thai, Japanese and Romanian (e.g., [21]-[24]), and its use as a real and efficient tool in natural language processing (e.g., [25]), one can claim that MG can be regarded as a one way out of two current challenges of Arabic, which are syntactic-semantic formalization and computation on logical or non-statistical level, at the same time.

The current paper is divided into two main sections. In section one, I offer a formalization of a small fragment of Arabic quantification by providing a Montagovian syntactic calculus and then formal interpretation of the fragment. The process of formalization is illustrated by using some random Arabic quantified expressions. In section two, I proceed to explain how Arabic can be amenable to logical computation via such formalization.

II. FORMALIZATION OF A FRAGMENT OF ARABIC

A. The Syntax of the Fragment

As many commentators (e.g. [26]-[29]) have noted, determining a system of syntactic derivation is the first step towards a Montagovian formalization of a fragment of a language. In what fellows, I adopt Montague’s method where the mathematics of Arabic sentence structure is defined in terms of syntactic categories and syntactic calculus for a fragment of Arabic quantification. Syntactic categories are divided into two types: primitive and complex categories. Table I below identifies both types of syntactic categories and syntactic calculus for a fragment of Arabic quantification. Syntactic calculus is formulated in terms of a categorial system. The syntactic component of MG is formulated indirectly in terms of model-theoretic semantics via an intermediate language of intensional logic.

In what fellows, I shall provide a syntactic calculus for some Arabic quantified expressions. The syntactic calculus below is based on MG in two respects: (a) Montague’s syntactic rules and (b) Montague’s method of syntactic derivation (i.e., tree analysis). The process of syntactic derivation in both examples (1) and (2) is provided using:
guidance table, and tree analysis, as following (note: read the expression in us using right-to-left direction).

<table>
<thead>
<tr>
<th>Category index</th>
<th>Index definition</th>
<th>Category name</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ</td>
<td>Primitive</td>
<td>Sentence</td>
</tr>
<tr>
<td>ε</td>
<td>Entity expression; or proper name</td>
<td></td>
</tr>
<tr>
<td>ḍ</td>
<td>t/IV</td>
<td>Intransitive verb phrase</td>
</tr>
<tr>
<td>e</td>
<td>IV/T</td>
<td>Transitive verb phrase</td>
</tr>
<tr>
<td>C</td>
<td>e/IV</td>
<td>Common noun phrase</td>
</tr>
<tr>
<td>DET</td>
<td>ḍ/CN</td>
<td>Determiner</td>
</tr>
<tr>
<td>CAN</td>
<td>CN/CN</td>
<td>Adjective</td>
</tr>
<tr>
<td>IV/t</td>
<td>IV/t</td>
<td>Sentence-taking verb phrase</td>
</tr>
<tr>
<td>IV/IV</td>
<td>IV/IV</td>
<td>IV-taking verb phrase</td>
</tr>
</tbody>
</table>

Example (2) is slightly complicated since it is constructed of seven syntactic categories [T, IV, TV, IV/IV, CN, DET, τ]. Therefore, in addition to the two known directions (i.e., bottom-up and left-to-right) a new direction will be involved in the process of syntactic derivation. The new direction is forward/backward, and will be illustrated throughout the process of syntactic derivation. To analyze Example (2) I start, as normal, with categorizing [rajul-i-n /man] as CN. Next, using S2 and forward direction, CN [rajul-i-n /man] combines with DET [kull-u /every] to produce T [rajul-i-n kull-u /every man]. Next, using S5 and backward direction illustrated by the arrow, T [ṭiib-a-ha] combines with TV [yu-hibb /love] to produce IV [yu-hibb-a-ha /love her]. Next, using S7 and forward direction, IV/IV [an ta-tamanna /wish to] combines with IV [yu-hibb-a-ha /love her] to produce IV [yu-hibb-a-ha an ta-tamanna /wish that love her]. Next, using S4, T [ṭiib/a] combines with IV [yu-hibb-a-ha an ta-tamanna /wish to love her] to produce a sentence τ [yu-hibb-a-ha an hia ta-tamanna /she wishes that every man loves her]. Then, using again S8, T [Sārah /Sarah] combines with τ [rajul-i-n kull-u /every man] combines with τ [yu-hibb-a-ha /wish to love her] to produce τ [rajul-i-n kull-u yu-hibb-a-ha an hia ta-tamanna /she wishes that every man loves her]. Then, using again S8, T [Sārah /Sarah] combines with τ [rajul-i-n kull-u yu-hibb-a-ha an hia ta-tamanna /she wishes that every man loves her] to produce τ [rajul-i-n kull-u yu-hibb-a-ha /Sarah /Sarah wishes that every man loves her].

Example (1) is a verbal sentence/jumla fi‘lIyya, whose initial word is an intransitive verb (IV) [hadar-a /come]. To analyze the sentence, I start, using the bottom-to-up and left-to-right directions as illustrated by arrows, with categorizing [at-tullāb-i /students] as CN. Next, using S2, CN [at-tullāb-i /students] combines with DET [jamī‘-u /all] to produce T [at-tullāb-i jamī‘-u /all students]. Then, using S4, T [at-tullāb-i jamī‘-u /all students] combines with IV [hadar-a /come] to produce τ [at-tullāb-i jamī‘-u ḍhadar-a /All students came] (shown in Table II, Fig. 1).

<table>
<thead>
<tr>
<th>Syntactic category</th>
<th>DET</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Express in use</td>
<td>at-tullāb-i jamī‘-u hadar-a</td>
<td></td>
</tr>
<tr>
<td>Morpheme-by-meaning gloss</td>
<td>DEF-student.PL -GEN all-NOM com. PST-3MS</td>
<td></td>
</tr>
<tr>
<td>Full translation</td>
<td>All students came.</td>
<td></td>
</tr>
</tbody>
</table>

B. Compositional Interpretation of the Fragment

Reference [29] shows, the most important features of MG are its use of model theoretic semantics which has commonly been used for the semantics of logical languages, and its adherence to the principle of compositionality—that is, the meaning of the whole is a function of the meanings of its parts and their mode of syntactic combination. I present below a compositional semantic interpretation of the fragment. Using Montague’s rule-to-rule translation method, I shall show how the above examples of Arabic quantified expressions can be translated into the language of intensional logic. The process of translation is illustrated by Fig. 3 and Fig. 4 below.
The most surprising outcome to emerge from this investigation is that MG is absolutely sufficient to provide a successful compositional interpretation to my fragment. That is, both procedures of syntactic derivation and semantic analysis of my fragment are in parallel with those used in MG. The results obtained from the preliminary syntactic and semantic analysis of my fragment have indicated that within the scope of my fragment no developments or extensions were required to the original theory.

The point to be made here is that, since the Montagovian framework is mainly based on analysis of English, which belongs to the Indo-European language family, success in applying the Montagovian framework to Arabic, which is Afro-Asiatic, was a 'challenge'. As it was expected, in the light of the differences between Arabic and English, some adjustment needs to be made to Montague's approach - whether in details or more widely - to accommodate features of Arabic generally, and Arabic quantification specifically. Contrary to expectations, this study did not find a significant difficulty in producing a compositional interpretation to Arabic, using Montague original theory of grammar, without the need to additional factors. In addition, this study did not find significant differences, in apparatus or process of application of MG, between Arabic and cases of other language groups (i.e., [22]-[24] above). This increases the evidence for the hypothesis, according to Farghaly [30, p.43], that Arabic shares many properties with other natural languages, and interesting insights can be gained by analyzing the structure of Arabic by any of the prevailing linguistic theories or formalisms.

However, with a limited portion or fragment of Arabic quantified expressions, caution must be applied, as the findings might not be transferable to all linguistic phenomena of Arabic, or even to all Arabic quantified expressions. More specifically, MG is limited to treat certain quantified expressions in Arabic for two respects:

1) Montague’s syntactic and semantic rules (as presented in Montague’s PTQ) are designed in a way that does not fit well certain expressions of Arabic quantification, however they do fit well other such expressions. To be exact, in addition to the noun phrase construction, Arabic quantifiers are used in further different constructions such as: (a) determiner and demonstrative [DET+DEM], (b) determiner and term (i.e., proper noun) [DET+T], and (c) determiner and pronoun [DET+PRO].

2) Certain characteristics of Arabic can be regarded as a source of limitation since certain quantified expressions in Arabic are used in grammatical forms that are out of the range of MG. To be exact, based on ([31]-[33]), Arabic quantifiers are classified into three classes, according to morphological and syntactic aspects: (1) nominal quantifiers, (2) numerals and (3) phrasal quantifiers. Furthermore, the class of nominal quantifiers in itself is classified into three sub-classes, according to the range of quantity: (i) expressions of totality, (ii) expressions of majority and (iii) expressions of partiality. In each of these classes there are a number of quantifiers which can not be represented in terms of Montague's rules.

Both limitations require the creature of new and specific rules to treat certain constructions of Arabic, or proceed to post-Montague semantics.

### III. TOWARDS A LOGICAL COMPUTATION OF ARABIC

In the field of applied logic, which concerns the uses of logical languages and methods in computer science, it has become increasingly difficult to ignore that logic does well enough in both programming and implementing several computational systems. That is, the second half of the 20th century witnessed an intensive interaction between logic and computer science, to the extent that computer scientists have utilized many logical languages and techniques in areas such as program verification, semantics of programming languages, automated theorem proving, and logic programming (see [34]-[37]). To be more exact, reference [38] shows, language (spoken and written) is central to all aspects of our communication. Therefore natural language processing systems, both current and future, are bound to play a crucial role in our communication with machines and even among ourselves [p. 1242]. Besides, there is no doubt that logic is considered an essential tool to construct these systems, and it is capable to do so because it has the formal notation, as reference [39] shows: "logic was developed as a formal
notation to capture the essential properties of natural language and reasoning” [p. 515].

To illustrate, among various logical methods and tools, categorial grammar (CG) can be considered the main representative of logical methods that have widely used to accomplish computational systems of natural languages on the logical or non-statistical level. Historically, as reference [40] shows, CG come of a tradition of linguistic description rooted in philosophy of language, logic and algebra [p.1]. CG is not a single grammar but it is divided to many sorts such as: basic categorial grammar or "Lambek" categorial grammar, combinatory categorial grammar (CCG), unification categorial grammar (UCG) and MG as an extension to CG. It is no coincidence that the first, abortive flowering of categorial grammar for serious natural language description came with the first attempts at implementing natural language processing on a computer [Ibid]. That is to say, researchers in NLP have implemented systems and programs which based on CGs (eg., CATLOG system, MCGTOOLS system, The Natural Deduction CG Parser, ROSETTA machine translation).

Based on the current application, since Arabic shares other natural languages the possibility of formalization on the level of CGs, one can expect that the computational results that based on CGs of such languages could be extended to Arabic. That is, many computational systems and programs have been implemented in languages such as English, Japanese and Thai due to their amenability to CGs formalization. As illustrated above, Arabic is amenable to formalization, using one of CG extensions which is MG, and this increases the evidence for the hypothesis that logical computation of Arabic is considerably achievable.

IV. CONCLUSION

In this paper I attempted to present some results of my view of a formalization for a fragment of Arabic quantification. Based on the applicability of formal methods to Arabic, I believe that the progress in the field of NLP, that has been made in the last years, is also applicable to Arabic with some modifications. This formalization is based on MG, and it has been successfully used in several NLP systems to achieve deep syntax semantic Analysis. Unfortunately there are still little works reported from the Arabic computational linguistic community for semantic construction and formalization of Arabic. I hope that this approach might be a further motivation to redirect research to modern semantic construction technologies for developing an adequate model of semantic processing for Arabic.

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REFERENCES

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